Report on the volatility of Bitcoin and Ethereum in 2017-2021 in the context of applying the Black-Scholes-Merton valuation model.

The cooperating team of advisors from the Poznan University of Economics was asked to examine the assumptions of Brightpool's business model, specifically to see if the Black-Scholes model can be applied to the valuation of orders on Brightpool DEX. In other words, whether the option pricing model used in traditional financial markets will work for pricing options for which cryptocurrencies are the underlying instrument.

Dr. Marcin Bartkowiak of the Institute of Computer Science and Quantitative Economics at the Poznan University of Economics studied two significant cryptocurrencies: Ethereum (ETH) and Bitcoin (BTC), over the period 2017-2021 (the period during which both currencies were in operation) for prices expressed in USD. The first phase of development – the phase of gaining trust with a small number of transactions and stable prices, noticeable for all major cryptocurrencies – was omitted.

This report is the fruit of collaboration between a group of excellent researchers from the University of Economics in Poznan, Poland. The team was headed by Dr. Jacek Mizerka, co-author of the paper "The role of Bitcoin on developed and emerging markets - on the basis of a Bitcoin users graph analysis", which was published in the prestigious journal Finance Research Letters. The team also includes Dr. Marcin Bartkowiak, who is involved in derivatives research and whose work has been published in numerous international journals, Dr. Aleksandra Rutkowska, who, in cooperation with UCL researchers, created the very first trading algorithm for the insurance market, and Dr. Bartosz Kabacinski, the organizer of international financial conferences.

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1. Introduction

The first concepts to create an electronic currency by using cryptography date back to as early as the 1980s[1]. The idea did not catch on because of technological limitations and a lack of broader interest. It was not until 2008 that the conditions were favourable for the development of cryptocurrencies. Properly developed IT infrastructure and the financial crisis, which shook the stability of many currencies, the dollar in particular, established a fertile ground for the project proposed in the article titled "*Bitcoin: A Peer-to-Peer Electronic Cash System*" [2]. It described the principles of operation of a cryptocurrency called Bitcoin, with transactions saved in the blockchain. Since then, several thousand solutions alternative to Bitcoin have been created, but it still remains the most popular cryptocurrency with about 40% share in the market capitalization of all cryptocurrencies. Ethereum is second in this ranking, with a share of about 20%[3].

Therefore, cryptocurrencies are still a relatively new financial instrument, which is characterised by high dynamics of quotations and frequent large changes in prices. The properties of cryptocurrency prices and rates of return differ from those of the major currency pairs. So it would be justified to ask whether standard valuation models can be applied for options in which cryptocurrencies are underlying instruments. In particular, whether it is reasonable to use the Black-Scholes-Merton model.

2. The subject and purpose of the study of Bitcoin and Ethereum volatility.

The study was conducted for two major cryptocurrencies: Ethereum (ETH) and Bitcoin (BTC) in the period 2017-2021 for prices expressed in USD. BTC and ETH were introduced in 2009 and 2015 respectively. The chosen period is when both currencies already functioned, omitting the first phase of development, noticeable for virtually all cryptocurrencies: the

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[1]Mukhopadhyay U., Skjellum A., Hambolu, O., Oakley J., Yu L. Brooks R.,, *A brief survey of Cryptocurrency systems,* (2016) 14th Annual Conference on Privacy, Security and Trust (PST), 2016, pp. 745-752, doi: 10.1109/PST.2016.7906988.

[2] Nakamoto, S. (2008*). Bitcoin: A peer-to-peer electronic cash system*. Decentralized Business Review, 21260.

[3] Data from https://pl.tradingview.com/markets/cryptocurrencies/global-charts/ (accessed on 14.01.2022)

phase of gaining trust. At this stage, the number of transactions and trading are low, whereas the prices are stable.

The purpose of the study was to establish the properties of rates of return, especially volatility. The Black-Scholes-Merton (BSM) model assumes that the prices of the underlying instrument have a log-normal distribution, which is equivalent to the normal logarithmic distribution for rates of returns. Furthermore, according to the model the volatility is constant over time. Numerous studies on rates of return in various types of instruments, such as currencies, indices and stocks, have shown that the assumptions of the BSM model are unrealistic. Leptokurtic and heavy-tailed distributions of rates of return as well as volatility clustering are common. Nevertheless, the BSM model has remained a widely applied tool for the valuation of European options. Investors have learned to adjust the prices derived from the model so as to take into account the empirical properties of distributions. Therefore, if only BTC and ETH rates of return have properties that do not derive from classic financial instruments, it would be reasonable to use the BSM model for the valuation of options for these cryptocurrencies.

3. Study results

The first part of the study was to examine the prices and rates of return (Charts 1 and 2). The dynamics of changes was very similar for both cryptocurrencies. The prices in 2021 were much higher than in the previous years, and both BTC and ETH recorded their historical highs in that year. On the other hand, daily rates of return were subject to high fluctuations throughout the studied period, with the most prominent decline of about 40% on 12 March 2020, which was associated with the spreading SARS-CoV-2 pandemic.

The analysis of ETH and BTC prices and rates of return has demonstrated that these values are characterised by greater dynamics than classic instruments. Daily rates of return of over a dozen or even several dozen percent occur much more often than in the case of major currency pairs, for example.

The next stage of the study concerns empirical distributions of logarithmic rates of return. This involved the generation of histograms and estimation of density functions (Charts 3-4). The examined rates or return do not have a normal distribution, the histograms and empirical density functions significantly differ from the theoretical density function of normal distribution (with the same mean and deviation as the studied returns), showing leptokurtic (concentration of values around the mean) and heavy-tailed distribution (more frequent extreme values than in normal distribution).

The lack of normal and heavy-tailed distributions are confirmed by the quantile-quantile plots (Charts 5-6), which demonstrate that the empirical quantiles distant from the mean value differ significantly from the theoretical quantiles typical of a normal distribution.

The third stage of the study deals with the volatility of logarithmic rates of return. Volatility is not directly observable on the market, and hence an estimator is required for its measurements. The volatility analysis can be conducted through two approaches:

- static
- dynamic.

In statistical (also known as historical) terms, volatility is estimated as an annualized standard deviation of the daily logarithmic returns, and this deviation is calculated for a fixed length of observation window.

Chart 1: ETH and BTC prices in the period 2017-2021

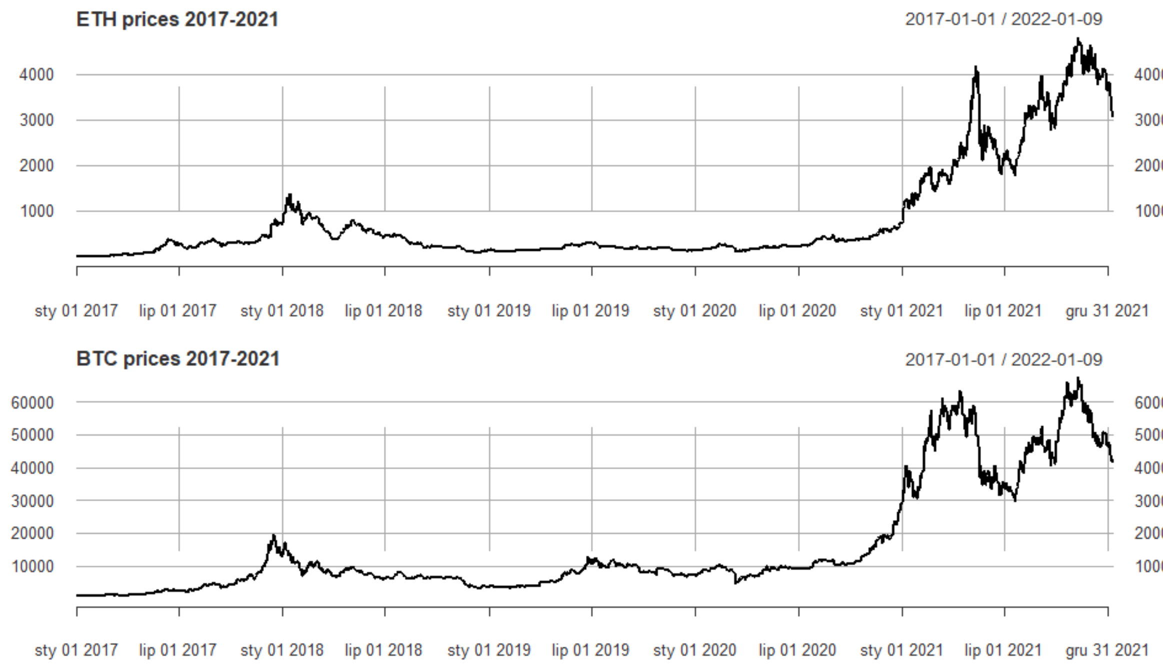


Chart 2: ETH and BTC rates of return in the period 2017-2021

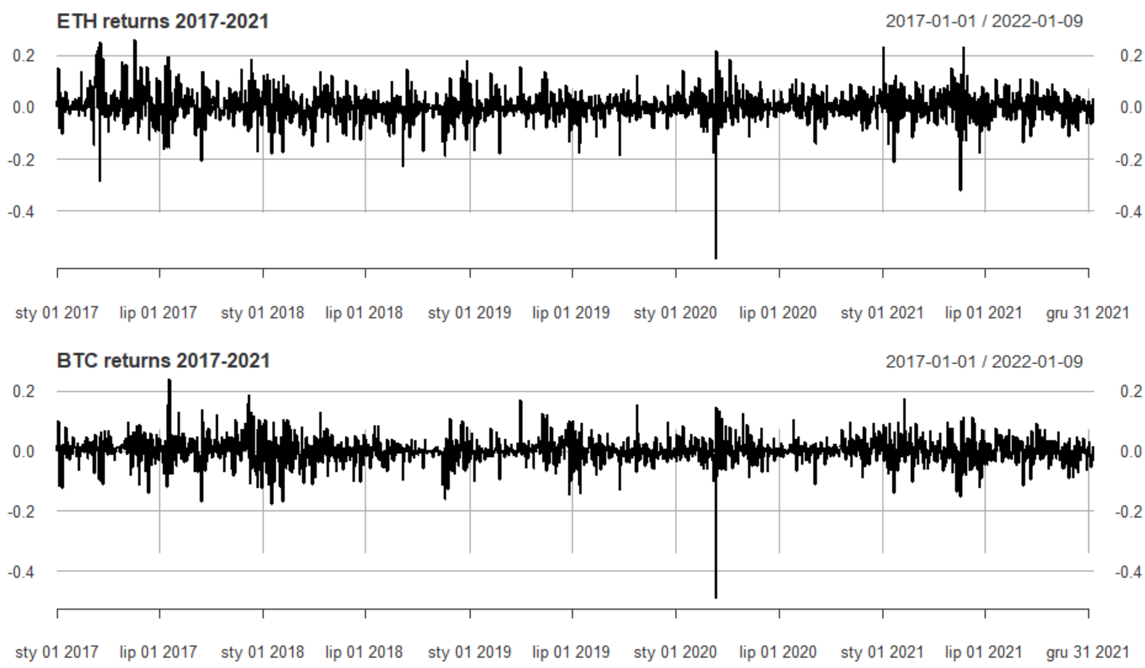


Chart 3: Histogram and empirical density function of ETH return rate distribution against normal distribution.

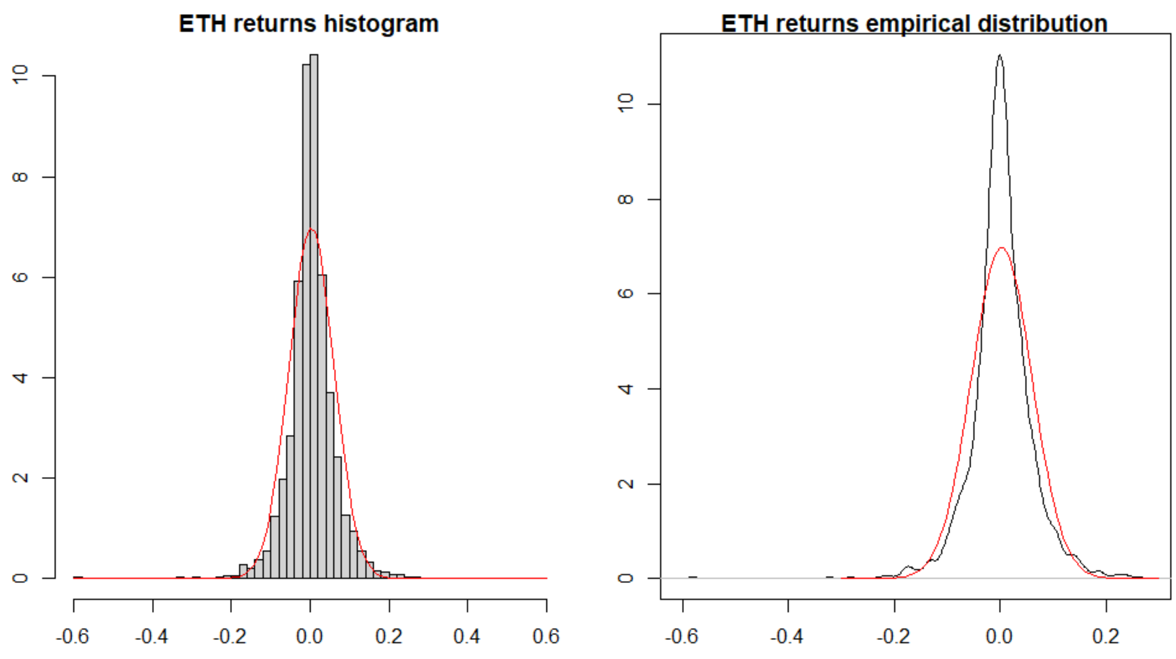


Chart 4: Histogram and empirical density function of ETH return rate distribution against normal distribution.

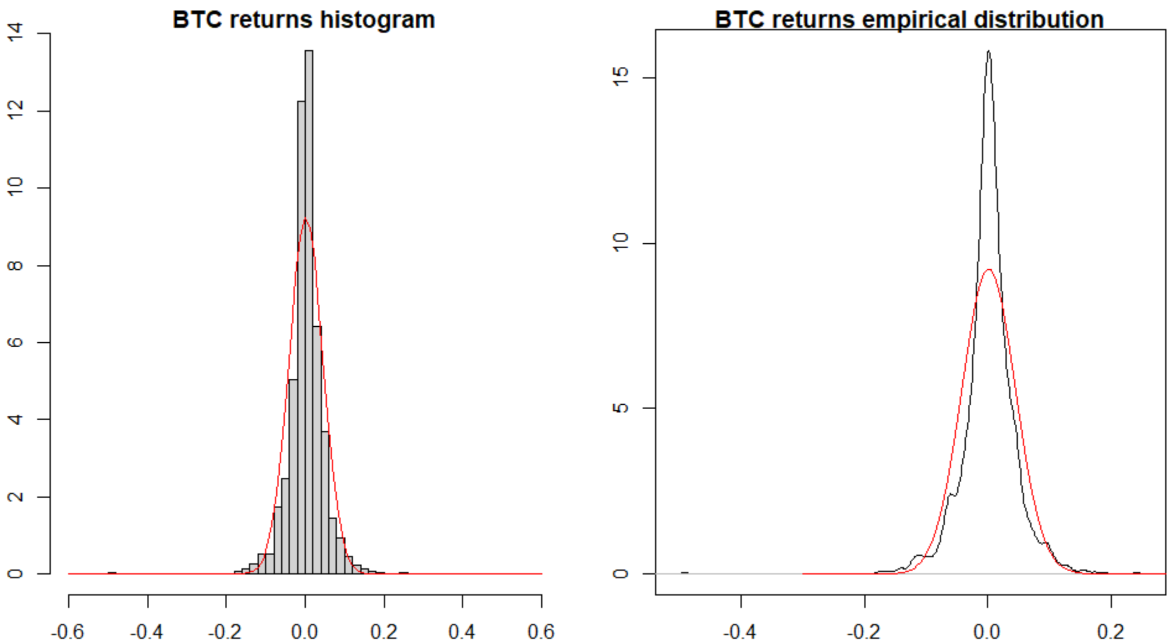


Chart 5: Quantile-quantile plot for ETH rates of return

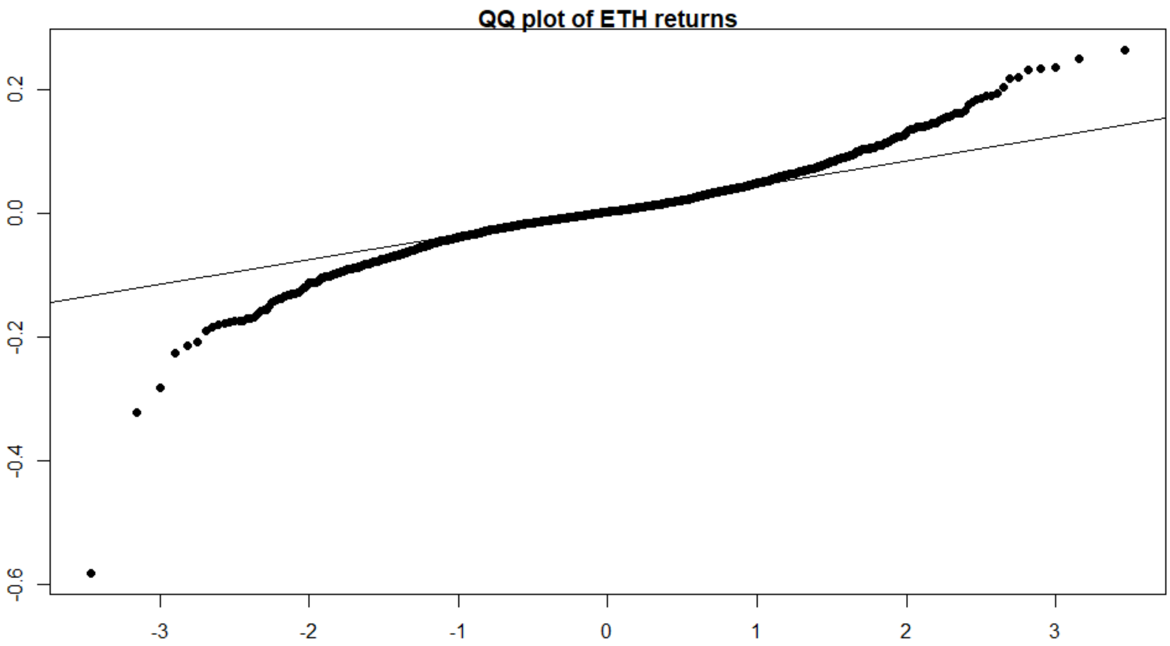
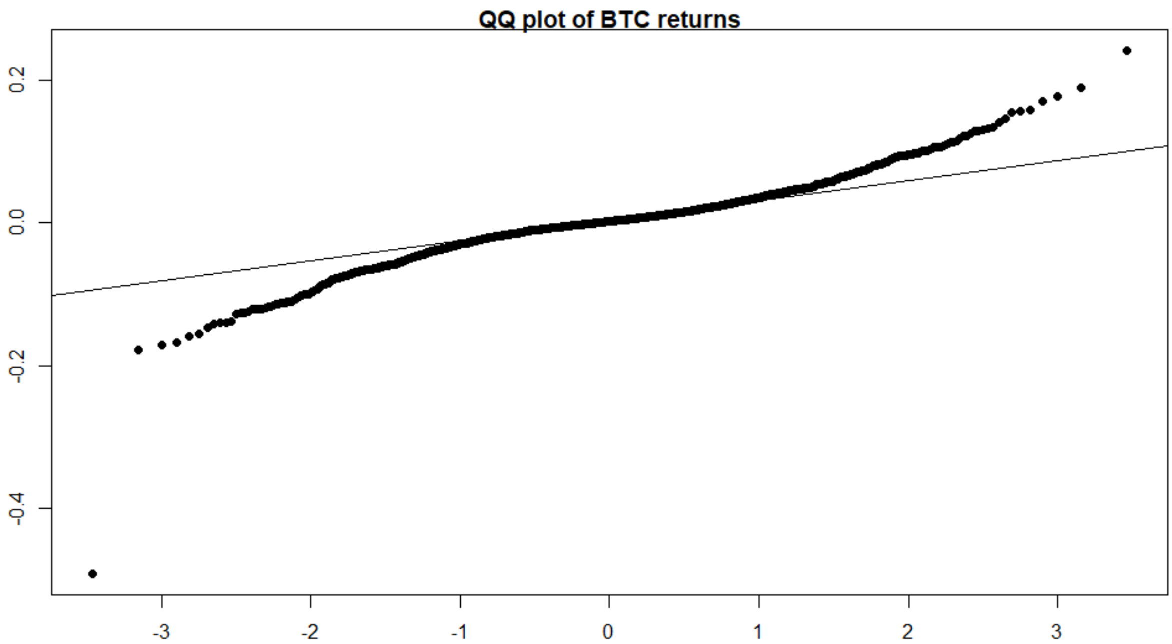


Chart 6: Quantile-quantile plot for BTC rates of return



It is tacitly assumed that the returns are independent or at least uncorrelated and have the same distribution on the length of the observation window.

Conditional distributions and conditional moments, in particular conditional variance (or conditional standard deviation), were examined in dynamic terms. These quantities are random variables measurable against the set generated by the incoming information on the price process. Therefore, the distributions of these variables can evolve over time, which means that new estimates of the distribution parameters must be calculated at each observation point on the basis of past observations. If we assume that current rates of returns are influenced by past returns and at the same time that future returns depend on current ones, it is necessary to use conditional distributions.

A conditional standard deviation is called conditional volatility. In practice, it is calculated by using stochastic volatility (SV) models or generalized autoregressive conditional heteroskedasticity (*GARCH*) models. The SV models are continuous time models, whereas the GARCH models are discrete time models.

Historical volatility was determined for observation windows of one year, one quarter, one month and one week. The volatility determined in this way is very high for both cryptocurrencies and, naturally, the shorter the window for which the standard deviations are determined, the higher that volatility is (Charts 7-10). Historical volatility for classic assets is much lower - a few to more than several percent for major currency pairs, and more than several to several dozen percent for stock exchange indices.

In the presented charts we can also see one of the greatest disadvantages of historical volatility: a single extreme observation may inflate the volatility for a period equal to the length of the observation window. This is exactly what happened in 2020, when the aforesaid March decline increased the volatility to a much higher level, which was then maintained until the extreme observation left the observation window. For this reason, the study of the nature of volatility should be supplemented with a dynamic approach. For this purpose, the ARMA(1,1)-GARCH (1,1) model was estimated as the basis for determining the conditional volatility The simplest specification of the GARCH model was chosen since numerous studies show that such model can be estimated with relative ease, and at the same time it allows us to capture non-linear dependencies in conditional variance. Such volatility is characterised by high dynamics (Charts 11-12), and periods of low volatility are interspersed with periods of high volatility, while the frequency of these changes is high and there are no long periods of relatively stable volatility.

One of useful parameters when studying conditional volatility is the so-called persistence, which informs about how quickly large volatilities disappear after a shock. Persistence can be expressed as the so-called half-life.

Chart 7: ETH and BTC historical volatility in the period 2017-2021 (annual window)

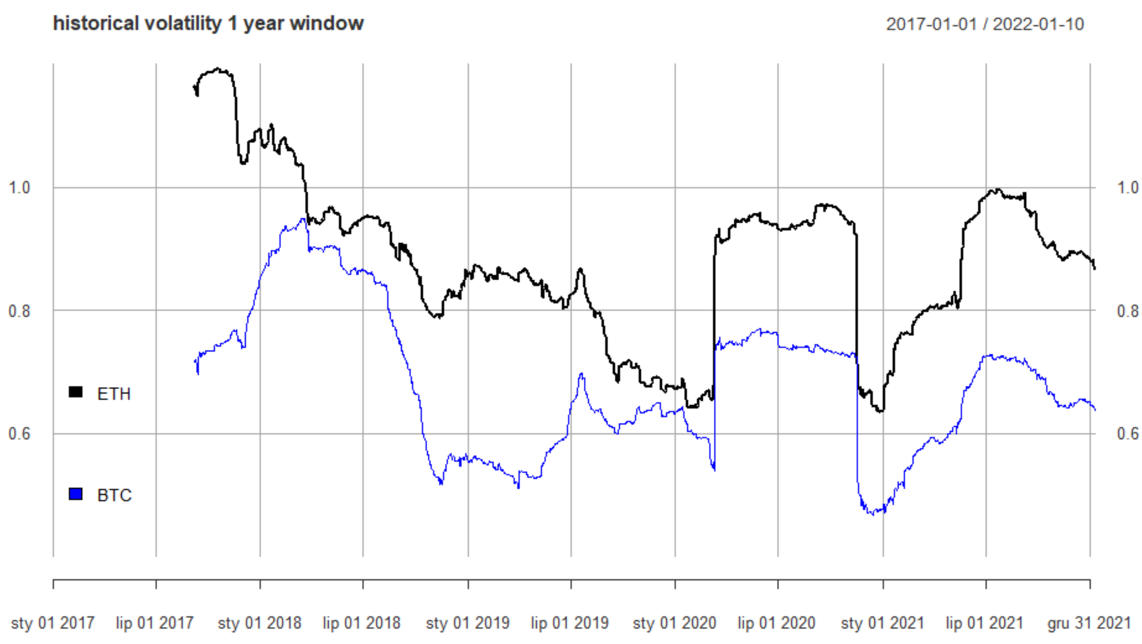


Chart 8: ETH and BTC historical volatility in the period 2017-2021 (quarterly window)

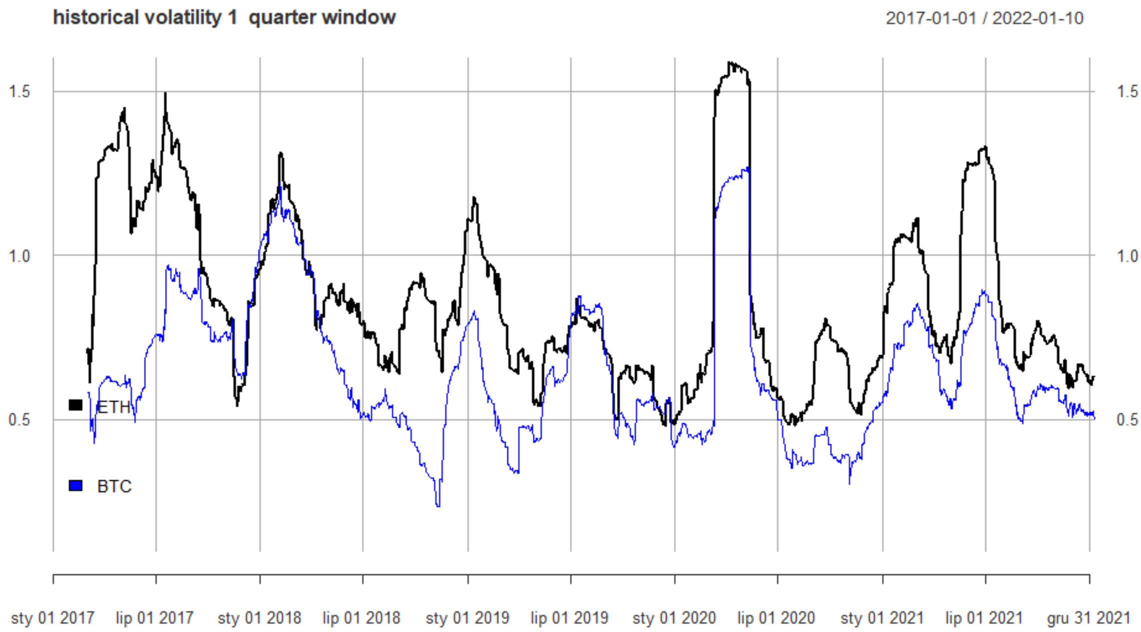


Chart 9: ETH and BTC historical volatility in the period 2017-2021 (monthly window)

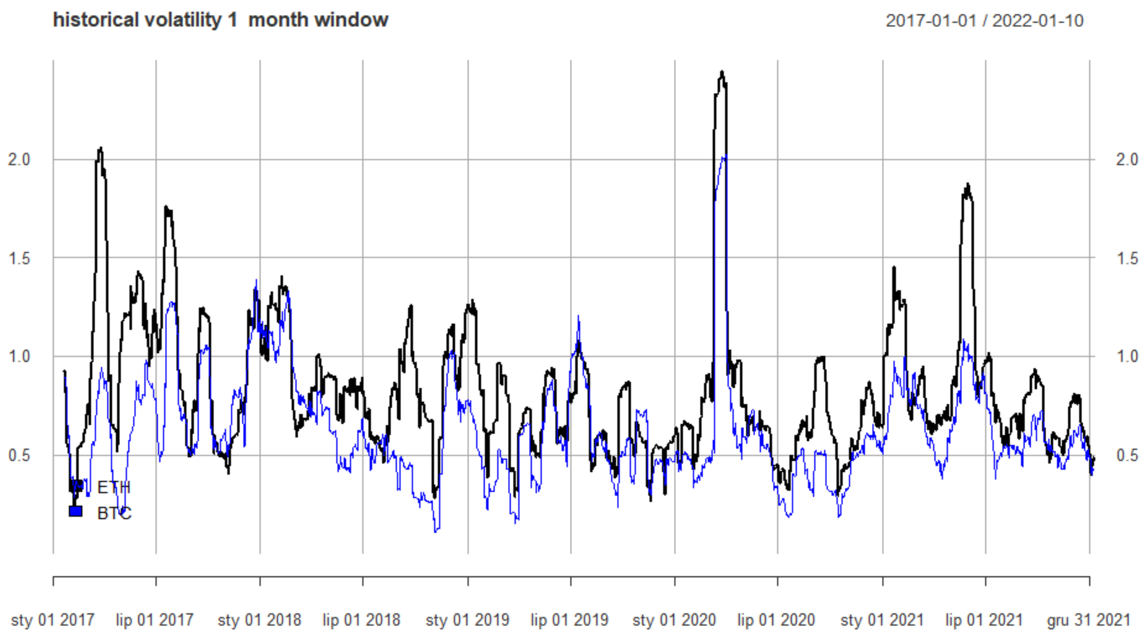


Chart 10: ETH and BTC historical volatility in the period 2017-2021 (weekly window)

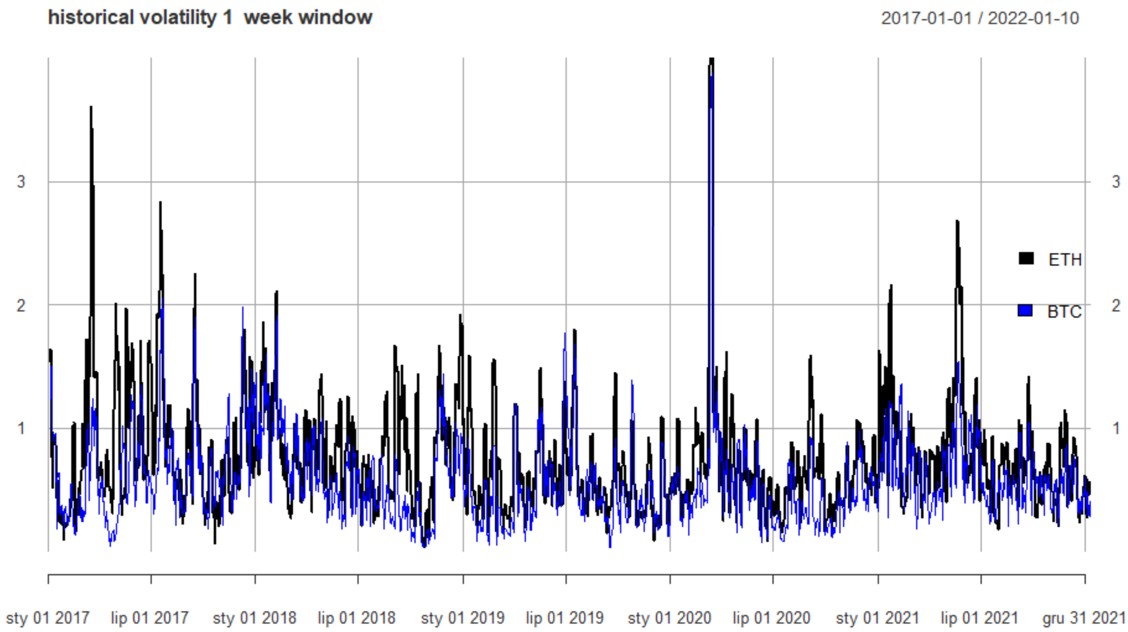


Chart 11: Conditional volatility and daily returns for ETH

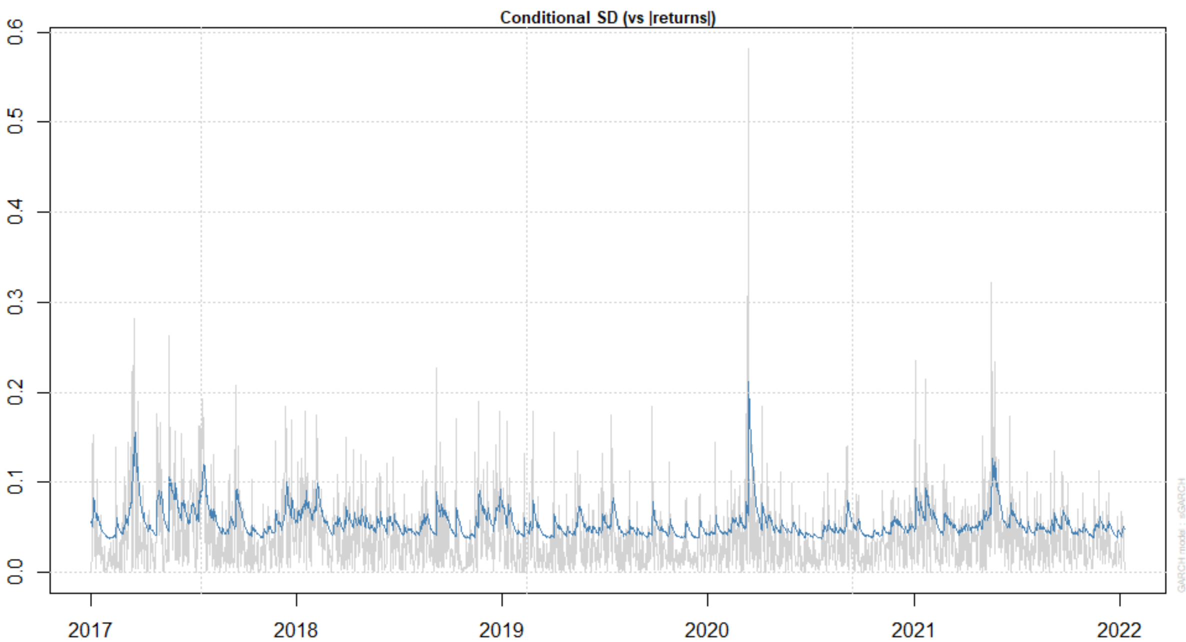
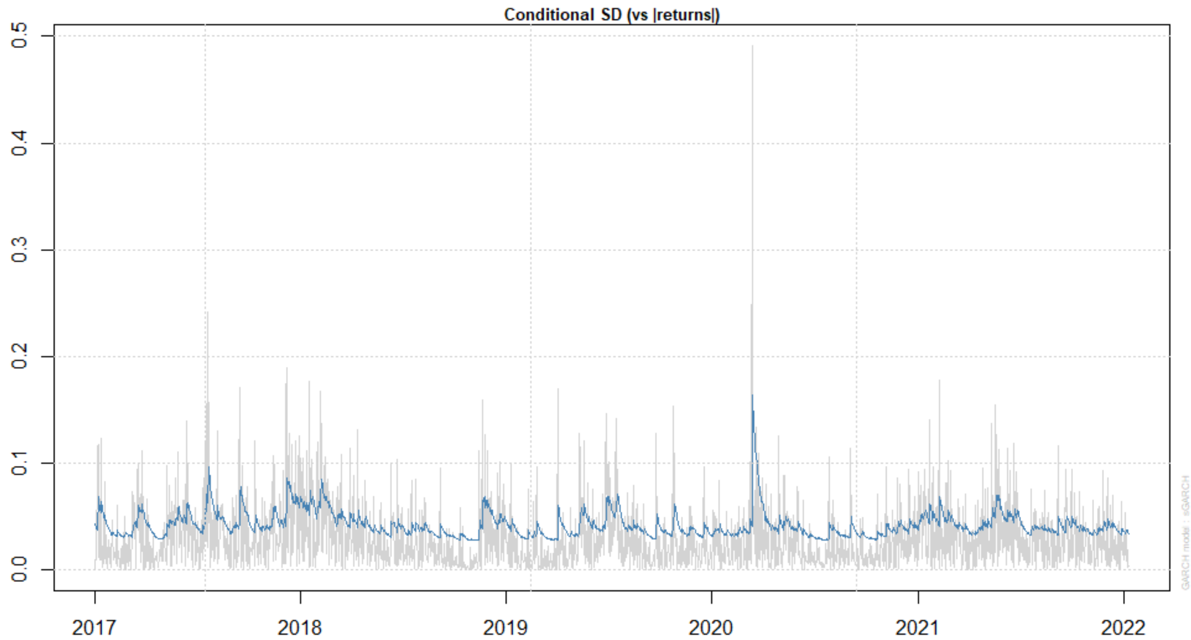


Chart 12: Conditional volatility and daily returns for BTC



For GARCH (1,1) model, the half-life is:

$$\frac{\ln ln\ 0{,}5}{\ln ln\ (\alpha_1 + \beta_1)}$$

where α_1 and β_1 are parameters of the GARCH(1,1) model and the unit is the length of the interval for which returns are calculated (daily returns are used in the study). The estimated parameters and half-lives for ETH and BTC are listed in Table 1. The average half-life of high volatility is quite short, about 9 days for ETH and about 12 days for BTC, which confirms the high dynamics of volatility.

Table 1. Half-lives for ETH and BTC estimated using the GARCH(1,1) model.

	ETH	**BTC**
α_1	$0{,}1212$	0.1035
$\beta_!$	0.8053	0.8418
half-life	$9{,}0737$	12.3333

A switching GARCH model with two states (regimes) was used to check whether periods of low and high volatility can be identified in a given period. In this type of models, volatility is described by GARCH models with different parameters. One specification applies to a low-volatility regime, and the other to a high-volatility regime. Switching the model specification provides a more detailed description of the dynamics of volatility. In addition, this approach makes it possible to estimate the probabilities of changing or maintaining the current regime. In both cases, changes of state are very frequent for ETH and BTC, as shown in Charts 13-14; conditional probabilities of changing the current state are subject to fluctuations, so it is difficult to distinguish periods of greater or lower volatility.

The last stage of the study involved checking the longest period in which conditional volatility was maintained below and above a specified point. Quantiles of 0.1-0.9 with a step of 0.1 were chosen as limit values. The results are presented in Table 2. The longest periods of low volatility are short for both cryptocurrencies. For example, the longest period for a volatility below a quantile of 0.1 is 16 days for ETH and 29 days for BTC. At the same time, it should be borne in mind that a quantile of 0.1 means annual volatility of over 67% for ETH, and almost 55% for BTC, which is a few or even more than several times greater than the volatility observed for traditional assets. Yet, the longest period with volatility above a quantile of 0.1 for both currencies was longer than a year: 372 and 483 days for ETH and BTC, respectively. It is worth noting the disproportion between the longest periods of volatility above and below the median. For ETH, the longest period was 51 days for volatility below the median and 87 days for volatility above the median, while for BTC it was 45 and 179 days, respectively.

Chart 13: The probability of change in the current state and the value of conditional volatility for ETH.

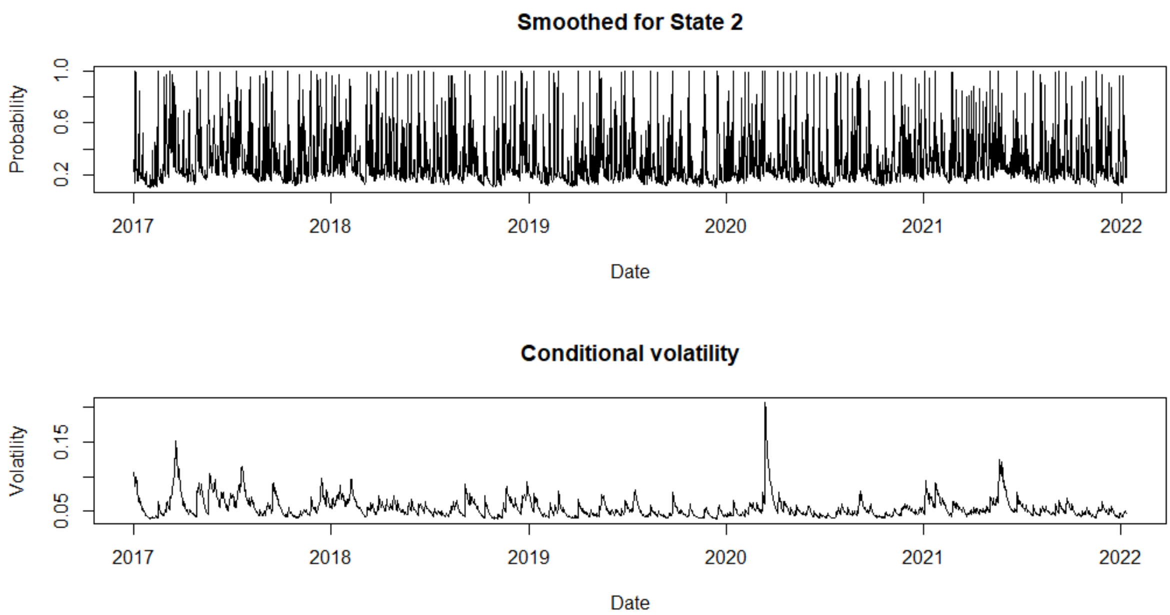
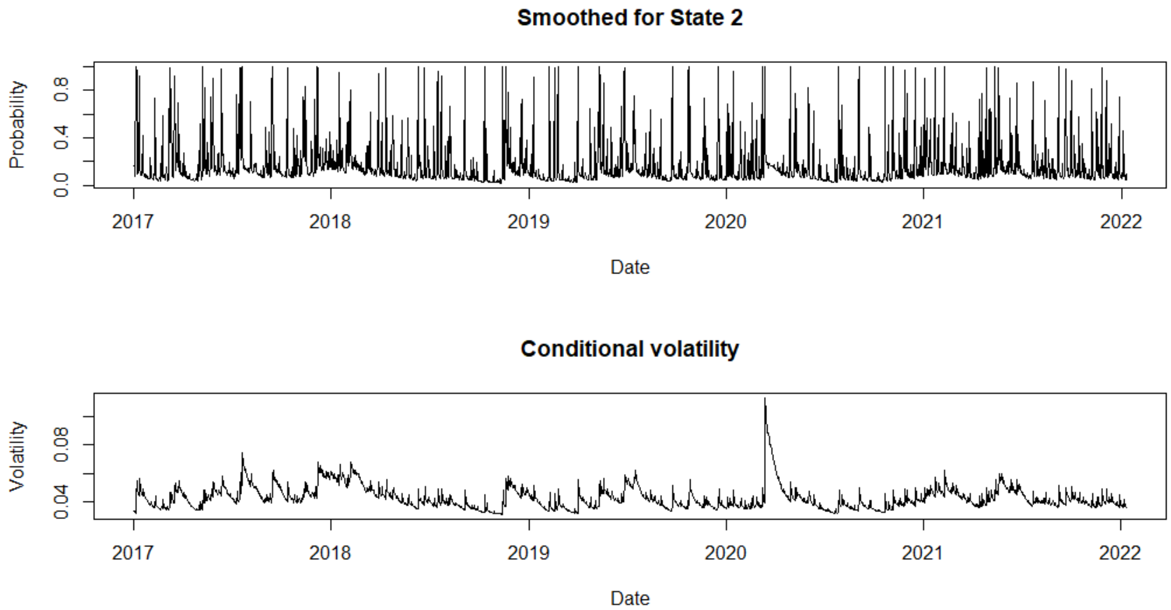


Chart 14: The probability of change in the current state and the value of conditional volatility fo



r BTC.

Table 2. The longest period of conditional volatility persisting below and above the specified limit value for ETH and BTC.

	ETH			BTC		
quantile range	quantile	Maximum period of volatility (in days)		quantile	Maximum period of volatility (in days)	
		below the quantile	above the quantile		below the quantile	above the quantile
0, 1	0.0426 (0.6737)	16	372	0.0345 (0.5461)	29	483
0, 2	0.0446 (0.7053)	23	118	0.0362 (0.5737)	32	391
0.3	0.0465 (0.7356)	25	95	0.0376 (0.5948)	34	376
0, 4	0.0487 (0.7703)	40	88	0.0392 (0.6193)	44	192
0, 5	0.0510 (0.8068)	51	87	0.0406 (0.6412)	45	179
0, 6	0.0536 (0.8482)	52	78	0.0425 (0.6720)	47	112

$0,7$	0.0572 (0.9038)	60	36	0.0452 (0.7142)	69	106
$0,8$	0.0626 (0.9895)	137	24	0.0480 (0.7588)	77	88
$0,9$	0.0724 (1.1450)	203	19	0.0530 (0.8387)	266	40

The annualized values of volatility are given in parentheses.

4. Summary

In the analysed period, the logarithmic rates of return for ETH and BTC did not have a normal distribution and were characterised by high volatility. This conclusion is not surprising, as it is typical of most financial instruments. However, heavy-tailed and leptokurtic distributions as well as the dynamics of volatility are even more exposed in cryptocurrencies compared to classic instruments. Because of these features the BSM model might underestimate the value of deep-out-of-the-money options and cause major changes in option valuation over short time intervals. However, as Brightpool position is getting closer to a long straddle position, the use of the BSM model does not generate any business risk. It should be emphasized that the long straddle is a strategy that allows us to take profit in the event of high volatility, when the absolute values of price changes are higher than option premiums. There is no guarantee that the high volatility of BTC and ETH will continue in the future, but there are currently no indications that this feature will disappear.